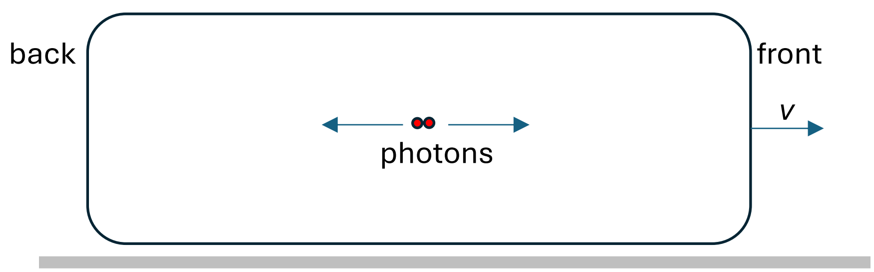
Teacher notes
Topic A

Galileo versus Einstein and simultaneity

A rocket of proper length L moves with speed v relative to the ground. Two photons are emitted from the middle of the rocket and are received at the front and back walls of the rocket.

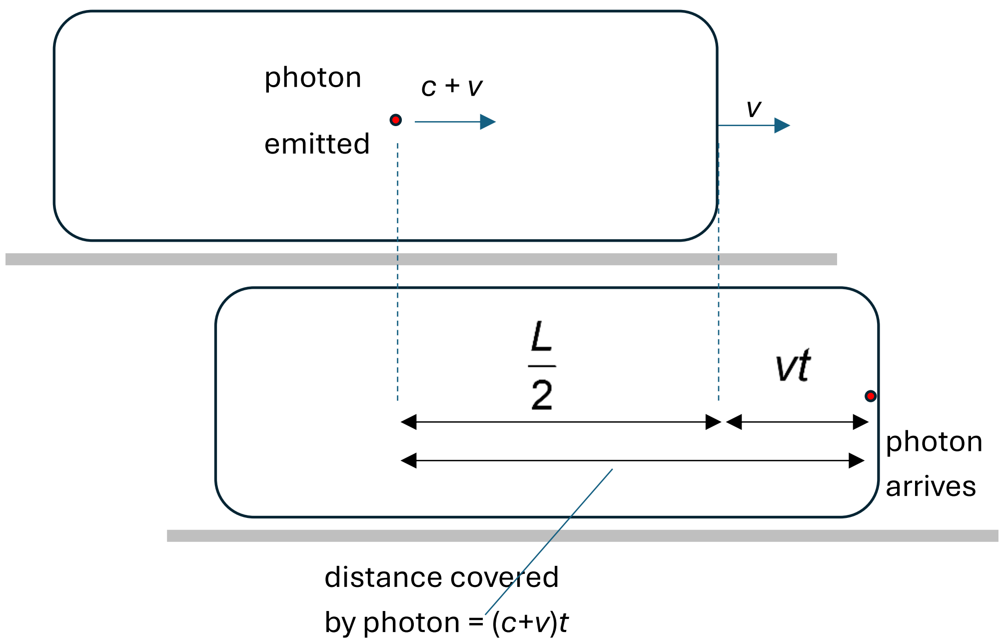


(a) Show that the photons arrive at the walls at the same time according to an observer in the rocket.
(b) Determine the arrival times of the photons at the walls according to an observer on the ground **using Galilean relativity**.
(c) Repeat (b) according to Einstein's relativity.

Answers

(a) Both photons travel a distance $\frac{L}{2}$ at speed c and so the time taken for both is $\frac{L}{2c}$. The photons arrive at the walls simultaneously.

(b) According to Galilean relativity the ground observer measures that the forward moving photon has speed $c + v$ and the backward moving photon has speed $c - v$. Let us look at the forward moving photon:



The diagram shows that $(c+v)t = \frac{L}{2} + vt$ (remember in Galilean relativity the speed of the photon is $c + v$ according to the observer on the ground). Hence, we find that

$$t = \frac{L}{2c}.$$

Similarly for the backward moving photon we find in the same way $t = \frac{L}{2c}$. These should not be surprising results: time is absolute in Galilean relativity and the rocket and ground observers agree on their time measurements. The photons arrive simultaneously for both observers.

(c) We know that things will be different in Einstein's relativity. The photons move with speed c for all observers. The arrival of the photons is simultaneous for the rocket observer but the arrivals happen at different points in space so they will not be simultaneous for the ground observer. The back wall is coming at the backward moving photon and so this photon will arrive first.

For the forward moving photon:

$$\Delta t_{\mathrm{F}} = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)$$

with $\Delta t' = \dfrac{L}{2c}$ and $\Delta x' = \dfrac{L}{2}$. This gives the answer

$$\Delta t_{\mathrm{F}} = \gamma\left(\frac{L}{2c} + \frac{v}{c^2}\frac{L}{2}\right) = \gamma\frac{L}{2c}(1+\frac{v}{c})$$

Similarly for the backward moving photon

$$\Delta t_{\mathrm{B}} = \gamma\left(\frac{L}{2c} - \frac{v}{c^2}\frac{L}{2}\right) = \gamma\frac{L}{2c}(1-\frac{v}{c})$$

The backward photon arrives a time $\Delta t = 2\gamma\dfrac{Lv}{2c^2} = \gamma\dfrac{Lv}{c^2}$ **before** the forward photon.